Symmetric upwind scheme for discrete Weighted Total Variation *

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introduction In this project, we were interested in the discrete formulation of Total Variation (TV) in image processing and computer vision as a prior for piecewise constant images. This topic has been widely studied since it has been proposed by Rudin-Osher-Fatemi (ROF) [1] and remains an active research field ; as an example, non-local formulations have been proposed in [2], [3] and a combination of TV with Non-Local means has been proposed in [4], all in order to preserve textures and thin structures. The Total-Generalized-Variation (TGV) studied in [5] and the method in [6] are variants that reduce the staircasing effect observed with TV.

Total Variation is applied in many imaging and computer vision problems such as segmentation [7], inpainting, deconvolution and motion estimation [8], optical flow [9], filtering [10], point cloud processing [11] and classification [12]. Most studies are based on continuous formulations that are then approximated to solve the discrete problem. In this work, we considered directly the discrete setting instead.

Motivation In [13], a new discrete TV formulation is proposed, referred to in the following as TV_u, and inspired by upwind finite difference schemes used in the numerical resolution of Partial Differential Equations. Despite the upwind scheme [13] ability to better preserve isotropy in comparison with the usual ones, Condat shows in [14] that it is not invariant to contrast inversion. Consequently, some artifacts tends to appear with TV_u such as light isolated dots, which are not suitable especially for filtering purpose. Condat also proposes a new dual formulation of the Total Variation, denoted here by TV_c, which avoids this issue. However, the price to pay is that this formulation is not explicit. Note in addition that both schemes do not check the discrete co-area formula. This formula, verified by the usual anisotropic TV definition, allows to solve some non-convex problems by convex relaxation such as segmentation, see for example [7].

Contributions In this project, we studied two non-local TV formulations on graphs. This type of graph-based formulation was initially proposed in [15]. The proposed formulations, referred to as $\mathsf{TV}^{\pm}_{\omega|p}$, rely on a symmetric definition combining an upwind scheme with a downwind scheme expressed in an ℓ^∞ and ℓ^2 norms respectively. We proposed an efficient

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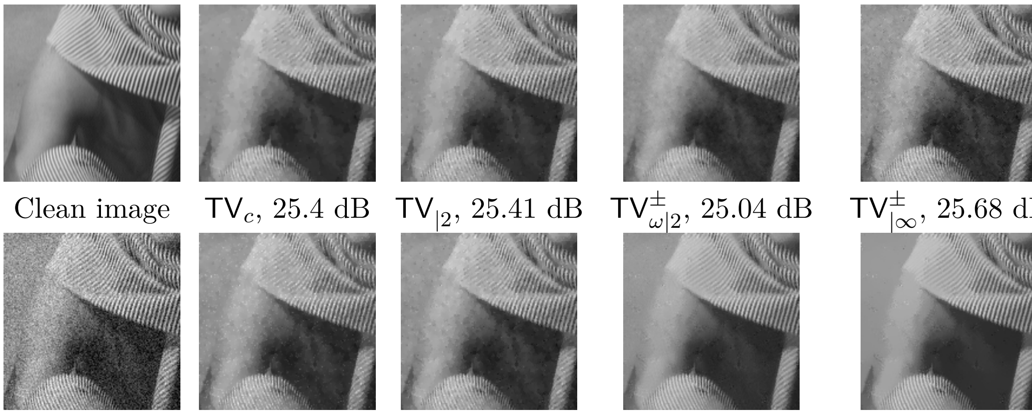
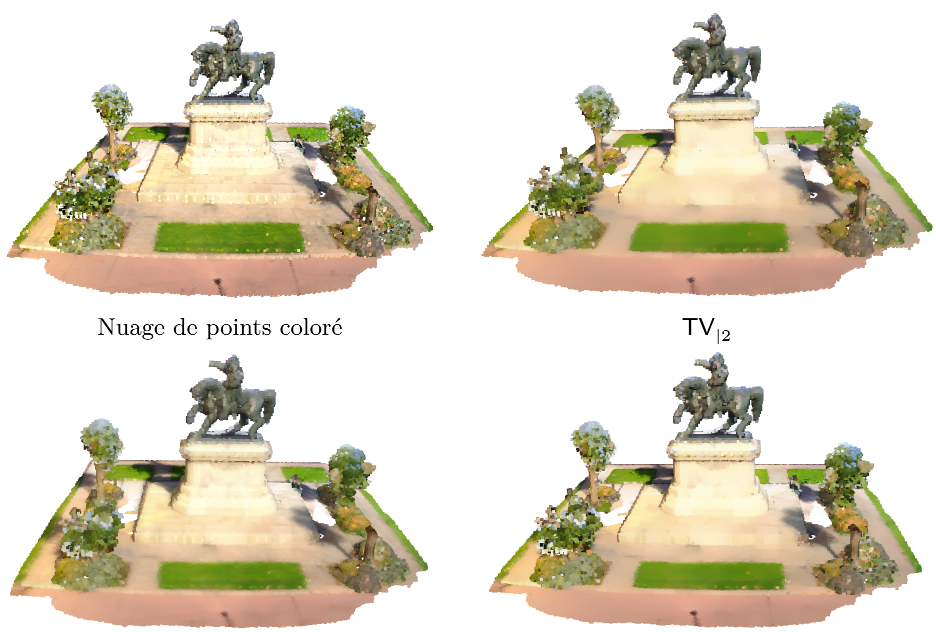


Clean image | TV_c, 25.4 dB | $\mathsf{TV}_{|2}$, 25.41 dB | $\mathsf{TV}^{\pm}_{\omega|2}$, 25.04 dB | $\mathsf{TV}^{\pm}_{|\infty}$, 25.68 dB

Noisy image | TV_u, 25.55 dB | $\mathsf{TV}^{\pm}_{|2}$, 25.76 dB | $\mathsf{TV}^{\pm}_{\mathrm{NL}|\mathrm{H}}$, 27.37 dB | $\mathsf{TV}^{\pm}_{\mathrm{NL}|2}$, 27.87 dB

FIGURE 1 – Denoising results with Gaussian noise ($\sigma = 20$). The proposed symmetric formulations do not suffer from artefacts, unlike the upwind scheme TV_u, which results in a better PSNR especially with the Non-Local (NL) weight computed computed as in [16]. Staircasing effect is reduced by using $\mathsf{TV}_{|\mathrm{H}}$, the Huber TV, see for instance [17].

way to express these norms allowing us to solve convex optimization problems with these regularization terms and illustrate their properties on some numerical experiments. These properties are : isotropy preservation, bias reduction in comparison with TV_u, better filtering results in comparison with variational approaches of the same type and the versatility of the non-local ℓ^∞ formulation that can mimic other formulations. An illustration of those properties is given in Figure 1, and an application to point-cloud processing in Figure 2.



Nuage de points coloré | $\mathsf{TV}_{|2}$

$\mathsf{TV}^{-}_{|2}$ | $\mathsf{TV}^{\pm}_{|2}$

FIGURE 2 – Illustration of color image processing on point-clouds.

Publications and code source This works has been published in two peer-reviewed conferences proceedings [18, 19], that are available on HAL open archive. Source codes (in matlab script) are also available at https://sonia.wp.imt.fr/publications/.

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p-Laplacians on directed graphs for graph signal processing *

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1 Introduction

The graph Laplacian plays an important role in describing the structure of a graph signal from weights that measure the similarity between the vertices of the graph. In the literature, three definitions of the graph Laplacian have been considered for undirected graphs : the combinatorial, the normalized and the random-walk Laplacians. Moreover, a nonlinear extension of the Laplacian, called the p-Laplacian, has been put forward for undirected graphs. In this project, we have proposed several formulations for p-Laplacians on directed graphs that are inspired from the Laplacians on undirected graphs. Then, we consider the problem of p-Laplacian regularization of graph signals. Finally, we provide experimental results to illustrate the effect of the proposed p-laplacians on a specific type of graph signals (3D colored meshes).

2 Contributions

Given directed graphs, we consider three possible directed difference operators that are inspired from their counterparts on undirected graphs for the definition of the Laplacian. This corresponds to

$$(d_w f)(v_i, v_j) = w(v_i, v_j)(f(v_j) - f(v_i)) \tag{1}$$

$$(d_w f)(v_i, v_j) = w(v_i, v_j)\left(\frac{f(v_j)}{\sqrt{\mathrm{d}^-(v_j)}} - \frac{f(v_i)}{\sqrt{\mathrm{d}^+(v_i)}}\right) \tag{2}$$

$$(d_w f)(v_i, v_j) = \frac{w(v_i, v_j)}{\sqrt{\mathrm{d}^+(v_i)}}(f(v_j) - f(v_i)) \tag{3}$$

for combinatorial, normalized and random-walk difference operators with $\mathrm{d}^-(v_i)$ and $\mathrm{d}^+(v_i)$ the incoming and outcoming degrees of a vertex v_i.

Then the p-Laplacian of each of the aforementioned directed difference operators can be expressed as follows :

$$\begin{aligned}\Delta_w^{p,*} f(v_i) = \frac{1}{2}\Bigg(& f(v_i)\left(\sum_{v_j \to v_i} \frac{w(v_j, v_i)^2}{\phi(v_j, v_i)\|\nabla_w \vec{f}(v_j)\|_2^{2-p}} + \sum_{v_i \to v_j} \frac{w(v_i, v_j)^2}{\phi(v_i, v_j)\|\nabla_w \vec{f}(v_i)\|_2^{2-p}}\right) \\ & - \left(\sum_{v_j \to v_i} \frac{w(v_j, v_i)^2}{\gamma_1(v_j, v_i)\|\nabla_w \vec{f}(v_j)\|_2^{2-p}} f(v_j) + \sum_{v_i \to v_j} \frac{w(v_i, v_j)^2}{\gamma_2(v_i, v_j)\|\nabla_w \vec{f}(v_i)\|_2^{2-p}} f(v_j)\right)\Bigg)\end{aligned} \tag{4}$$

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with ϕ, γ_1 and γ_2 that are specific functions dedicated to each difference operator. From these p-Laplacian, we consider the following variational problem of p-Laplacian regularization on directed graphs :

$$g \approx \min_{f:\mathcal{V}\to\mathbb{R}} \left\{ E_w^{p,*}(f, f^0, \lambda) = \frac{1}{p} R_w^{p,*}(f) + \frac{\lambda}{2}\|f - f^0\|_2^2 \right\}, \tag{5}$$

which is related to the p-Laplacian since $R_w^{p,*}(f) = \langle \Delta_w^{p,*} f, f\rangle_{\mathcal{H}(\mathcal{V})} = \langle d_w f, d_w f\rangle_{\mathcal{H}(\mathcal{E})} = \sum_{v_i\in\mathcal{V}} \|(\nabla_w \vec{f})(v_i)\|_2^p$ and one can prove that $\frac{1}{p}\frac{\partial R_w^{p,*}}{\partial f(v_i)} = 2\Delta_w^{p,*} f(v_i)$.

Figure 1 presents an exemple of 3D colored mesh filtering. The considered graph is a symmetric directed mesh graph (provided from the scan) augmented with a 5-nearest neighbor graph within a 3-hop. To compare vertices we use a feature vector defined as the set of values obtained within a 1-hop . However, since the mesh graph is not regular, the feature vectors are not of the same size. We cannot use a L_2 distance to compare them, so we use instead the Earth Mover Distance between the histograms of the feature vector. For space contraints, we show results only with $\Delta_w^{p,rw}$ that provided the best results. With $p = 1$ the filtering enables a much better preservation of the signal sharp edges while removing noise.

FIGURE 1 – 3D colored mesh regularization on directed graphs (From left to right : original mesh, filtering with $\Delta_w^{p,rw}$, $\lambda = 0.05$, with $p = 2$ and $p = 1$).

3 Publications

This work has been published in two peer-reviewed conferences proceedings [1, 2], that are available on HAL open archive.

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Visual saliency and perceptual quality assessment of 3D colored meshes *

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1 Introduction

In every look thrown at a scene or an object, our attention is attracted by particular regions distinct from the surrounding zones. These striking areas, essentially prominent in the field of 3D objects, are content dependent. However, they are not dependent of the behavior or the experience relative to the human observer. Therefore, saliency computation can allow detecting perceptually important points or regions on the surface of a 3D mesh. Many applications in 3D computer vision benefit from visual saliency, we can mention : optimal view point selection where the objective is to generate the most informative viewpoints that capture a maximum of salient regions, and adaptive mesh simplification that aims at maintaining the best perceived quality by performing simplification essentially on regions of low saliency. In this project, we consider consider the notion of saliency for the processing and perceptual quality assessment of 3D colored meshes.

2 Contributions

In this project, we have proposed a novel multi-scale approach for detecting salient regions. To do so, we define a local surface descriptor based on patches of adaptive size and filled in with a local height field. The single-scale saliency of a vertex is defined as its degree measure in the mesh with edges weights computed from adaptive patch similarities weighted by the local curvature. Finally, the multi-scale saliency is defined as the average of single-scale saliencies weighted by their respective entropies (Figure 1).

We have used this saliency measure for optimal viewpoint computation (Figure 2).

However, this application rest on a partial estimation of visual saliency insofar that only geometric properties of the considered 3D mesh are taken into account leaving aside the colorimetric ones. As humans, our visual attention is sensitive to both geometric and colorimetric informations. Indeed, colorimetric information modifies the eye mouvements while visualizing a multimedia content. We have then proposed an innovative approach for the detection of global saliency that takes into account both geometric and colorimetric features of a 3D mesh simulating hence the Human Visual System (HVS). For this, we generate two multi-scale saliency maps based on local geometric and colorimetric patch descriptors. These saliency maps are pooled using the Evidence Theory. We have shown the contribution and the benefit of our proposed global saliency approach for two applications : automatic optimal viewpoint selection (Figure 3) and adaptive denoising of 3D colored meshes. Finally, the estimated visual saliency has been considered as a discriminative feature to assess the perceptual quality of 3D meshes. For colored meshes a dedicated 3D colored mesh database has been constituted.

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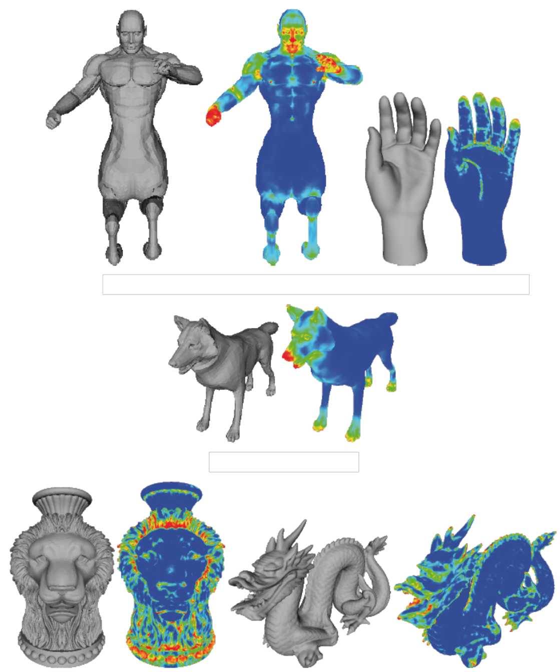

FIGURE 1 – 3D Saliency computation.

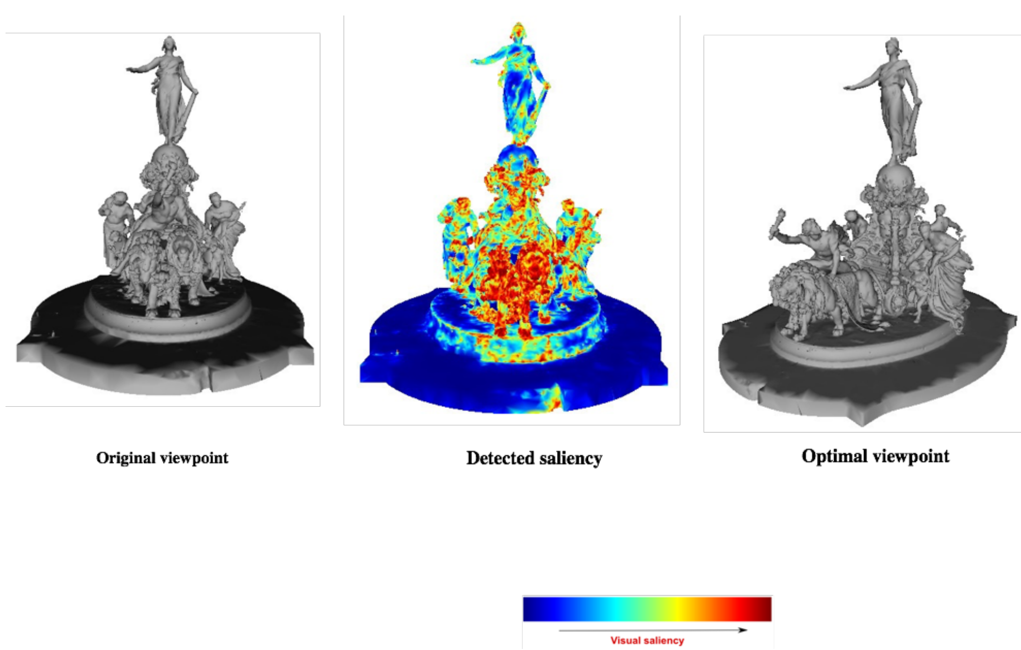

FIGURE 2 – Optimal viewpoint selection.

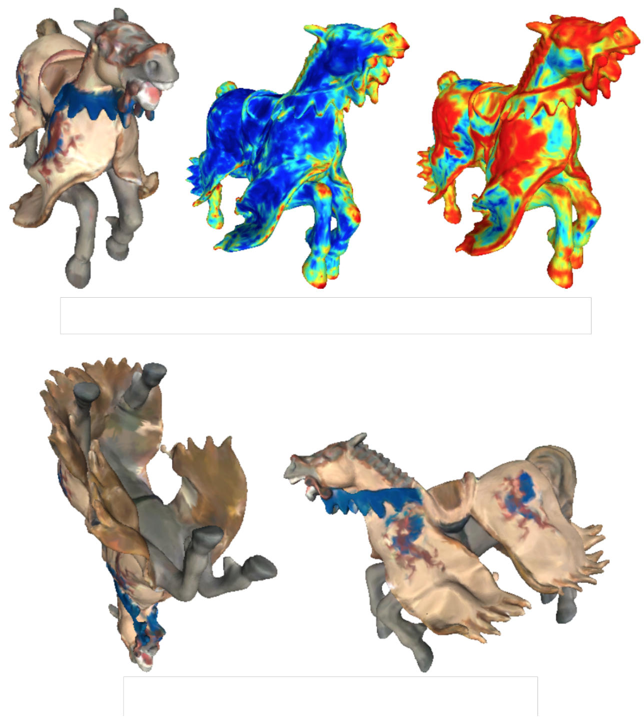

FIGURE 3 – Optimal viewpoint selection for colored meshes. First line, from left to right : original colored mesh, geometry-based saliency, the proposed global geometry and color saliency. Second line : Optimal viewpoint based on geometric saliency, optimal viewpoint based on global saliency.

3 Publications

This work has been published in four peer-reviewed conferences proceedings, one book chapter and a technical report [1, 2, 3, 4, 5, 6], that are available on HAL open archive.

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Multivariate morphological graph signal processing for 3D colored mesh editing*

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1 Introduction

Image filtering is recognized as one of the most important operation in image processing. In particular, structure-preserving filtering is an essential operation with a variety of applications in computational photography and image editing. During the last decade, a lot of structure-preserving smoothing filters have been proposed, and they aim at decomposing an image into prominent structures and fine-scale details, making it easier for subsequent image manipulation such as detail enhancement or visual abstraction. These filters were originally designed for images, but some have been extended to 3D meshes. In this project, we have proposed to investigate the use of Mathematical Morphology (MM) operators for such editing tasks. This has never been investigated before in literature.

2 Contributions

Morphological operators are non-linear vector-preserving filters (no new vectors are introduced in the processed image), and therefore they are not subject to the production of halos, which is a common problem in image editing tasks. The construction of morphological operators relies on complete lattices that impose the need of an ordering relationship between the elements to be processed. If MM is well defined for gray scale functions, there exists no general admitted extension that permits to perform morphological operations on vectors since there is no natural ordering of vectors.

We have proposed a framework for morphological processing of graph signals and investigated its usage for colored meshes editing tasks. The proposed method enables, with the help of the construction of a manifold-based ordering of color vectors, to define a new representation of graph signals in the form of an ordering of vectors and an index. The ordering relies on three steps : dictionary learning, manifold learning, and out of sample extension. This is illustrated by the Figures 1 and 2.

This new representation enables to formulate morphological operators for graphs signals and we have demonstrated the performance of the proposed method on various colored mesh editing applications (simplification, abstraction, enhancement). In particular we have proposed an approach for 3D mesh enhancement that decomposes a graphs signal into several layers (Figure 3) and recomposes them using a weighted recombination.

Each layer is manipulated by a nonlinear weighting function and a structure mask M (obtained by a saliency measure) that prevents boosting noise and artifacts while enhancing the main structures (Figure 4).

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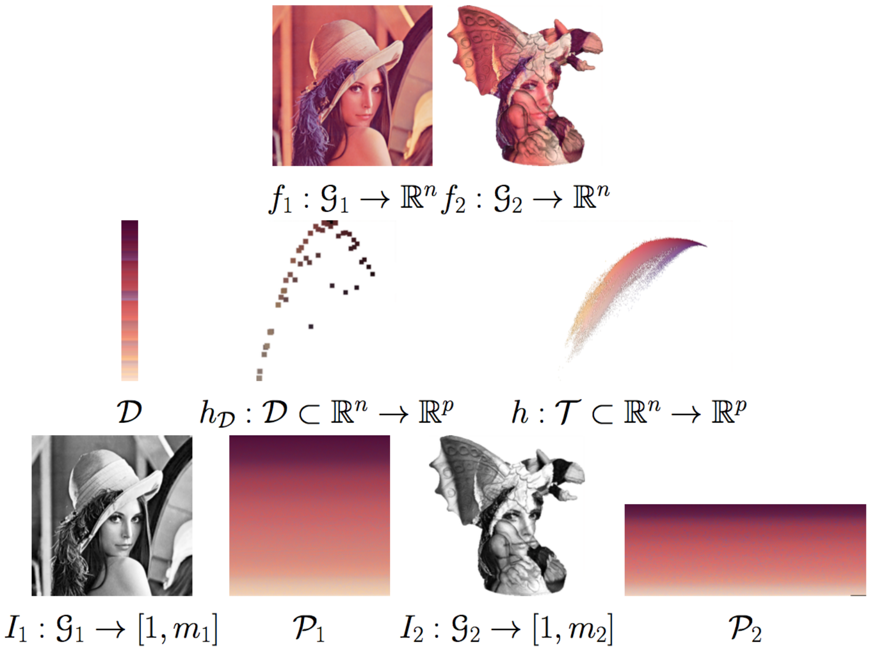

FIGURE 1 – Manifold-based vector-ordering.

FIGURE 2 – From left to right : a 3D colored graph signal f, and its representation in the form of an index graph signal I and associated sorted vectors $\mathcal{P}$.

FIGURE 3 – From top to bottom, left to right : an original mesh f, and its decomposition into three layers f_0, f_1, and d_1.

FIGURE 4 – Sharpening of a 3D colored mesh.

3 Publications

This work has been published in three peer-reviewed conferences proceedings, one international journal [1, 2, 3, 4], that are available on HAL open archive.

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